HEAT TRANSFER IN FILM BOILING ON A HORIZONTAL SURFACE

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Film boiling of a saturated liquid on a horizontal surface facing upwards is considered. An expression for the complex, determining the heat transfer rate, is obtained from the condition of interface stability loss; the relation is finalized by generalizing the experimental data. Two different mechanisms of bubble generation on the surface of a vapor film which account for the character of the derived relation are analyzed.

In addition to the problems arising in cryogenic engineering, in tempering processes, and in thermal power engineering, film boiling has also been considered in recent years in connection with emergency operating modes of water-moderated reactors. Here calculations need to be performed regarding a water feed to a horizontal heat sink surface heated to a high temperature.

In the majority of theoretical studies that deal with heat transfer of a saturated liquid on an extended horizontal surface facing upwards [1-6], the suggested relations are obtained on the basis of a hypothesis of a regular structure of the interface with space and time average cells of characteristic dimensions, and the generation of vapor bubbles is likened to a wave process with a certain characteristic wavelength. None of the above-stated works validate the adopted stationary model. Studies [1, 3, 4] solve motion and energy equations in one- and two-dimensional approximations for a "thin" laminar vapor film of constant thickness (a heating surface under the vapor bubble is regarded as thermally insulated). Characteristic dimensions of the cell and the bubble are specified from different considerations, but they are everywhere equal to or proportional to the Laplace constant.

In [5] the entire cell surface, including a region above the bubble, is assumed to transfer heat. In determining the dimensionless complex for subsequent data processing, the characteristic dimension is taken to be a thickness of the vapor film, its value being the same in both continuity and heat-conduction equations, although, taking into account its appreciable variation within the cell it would be logical to average thermal conductivity, rather than thermal resistance, in the heat-conduction equation.

Study [2] uses, with maximal simplifications, the same approach in principle as [1, 3, 4], with the difference that one of the examined models adopts the pressure loss by acceleration, instead of that by friction, as governing in the vapor motion equation.

Study [6] assumes that both laminar and turbulent flows can be implemented in a vapor film. Moreover, [6] allows for lateral vapor injection due to liquid evaporation at the interface. In lieu of the energy equation it utilizes a Reynolds analogy in the form derived for flow along a plate, without substantiating its applicability to a forced (in conformity with the adopted model) vapor motion in the vapor film. An assumption that the ratio of the characteristic dimension to the thickness of the vapor film is invariable is also employed without justification.

Two studies take the interface structure to be irregular, which makes the determination of the characteristic cell dimension unnecessary. Study [2] examines this hypothesis along with that on regularity of a structure of the vapor film surface. A relation for the Nusselt number at an irregular structure is derived from an appropriate relation relevant to the regular structure by changing an exponent of the governing complex so that the characteristic dimension is eliminated from an expression for the heat transfer coefficient.

Study [7] considers the problem as stationary with oscillatory character of the interface motion. Velocities in the vapor film are obtained from the simultaneous solution of the nonsteady heat-conduction equation and of the Navier–Stokes equations. Here, the motion equation takes into account only the gravitational component of the effective forces, without substantiating the disregard of others.

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TABLE 1. Relations for Heat Transfer in Film Boiling of a Saturated Liquid on a Horizontal Surface Facing Upwards

Literature citation	Relation	Equation number	Notes
[7]	$\mathrm{Nu} = 0{,}295\,(\mathrm{Ra}/K_{\mathrm{v}})^{1/3}$	(3)	
[1]	$\mathrm{Nu} = 0{,}425\,(\mathrm{Ra}/K'_{\mathrm{v}})^{1/4}$	(4)	
[2]	$\mathrm{Nu} = A_1\,(\mathrm{Ra}/K'_{\mathrm{v}})^{1/4}$	(5)	
[2]	$\mathrm{Nu} = A_2\,(\mathrm{Ra}\,\mathrm{Pr}_{\mathrm{v}}/K'^2_{\mathrm{v}})^{1/4}$	(6)	
[2]	$\mathrm{Nu} = A_3\,(\mathrm{Ra}/K'_{\mathrm{v}})^{1/3}$	(7)	
[2]	$\mathrm{Nu} = A_4\,(\mathrm{Ra}\,\mathrm{Pr}_{\mathrm{v}}/K'^2_{\mathrm{v}})^{1/3}$	(8)	
[3]	$\mathrm{Nu} = 0{,}41\,(\mathrm{Ra}/K'_{\mathrm{v}})^{1/4}$	(9)	$r' = r(1 + 0{,}95c_{p,\mathrm{v}}\theta_{\mathrm{w}}/r)$
[4]	$\mathrm{Nu} = A\,(\mathrm{Ra}/K'_{\mathrm{v}})^{1/4}$	(10)	
[8]	$\mathrm{Nu} = 0{,}012\,(\mathrm{Ra}/K'_{\mathrm{v}})^{1/2}$	(11)	$l = 2\pi\Lambda$
[5]	$\mathrm{Nu} = 185\,\mathrm{Pr}_{\mathrm{v}}K_{\mathrm{v}}^{0,09}$	(12)	$l = 2\pi\sqrt{6}\,\Lambda$, Physical properties of vapor on the saturation line
	for $\mathrm{Ar} < 10^8$		
[6]	$\mathrm{Nu} = 0{,}19\,\mathrm{Ar}^{1/3}\,\mathrm{Pr}_{\mathrm{v}}^{1/3}\,f_1(1/K_{\mathrm{v}})$ $f_1(1/K_{\mathrm{v}}) = \begin{cases} 1 & \text{for } 1/K_{\mathrm{v}} < 1{,}4, \\ 0{,}89(1/K_{\mathrm{v}})^{1/3} & \text{for } 1/K_{\mathrm{v}} > 1{,}4 \end{cases}$	(13)	$l = 2\pi\Lambda$
	for $\mathrm{Ar} > 10^8$		
[6]	$\mathrm{Nu} = 0{,}0086\,\mathrm{Ar}^{1/2}\,\mathrm{Pr}_{\mathrm{v}}^{1/3}\,f_2(1/K_{\mathrm{v}})$ $f_2(1/K_{\mathrm{v}}) = \begin{cases} 1 & \text{for } 1/K_{\mathrm{v}} < 2{,}0, \\ 0{,}71(1/K_{\mathrm{v}})^{1/2} & \text{for } 1/K_{\mathrm{v}} > 2{,}0 \end{cases}$	(14)	$l = 2\pi\Lambda$
[9]	$\mathrm{Nu} = 0{,}3\,(\mathrm{Ra}/K'^2_{\mathrm{v}})^{1/4}$	(15)	$r' = r(1 + 0{,}34c_{p,\mathrm{v}}\theta_{\mathrm{w}}/r^2)$

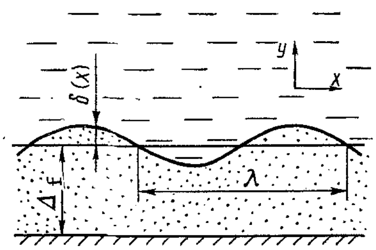

Fig. 1. Sketch of the process.

Table 1 presents, in chronological sequence, the relations obtained in the aforementioned studies and those derived in [8, 9] unifying the experimental data. In all cases, where it is not stipulated otherwise, physical properties of the vapor are taken at the mean vapor film temperature, and the expression for the evaporation heat, with a view to the vapor superheating, has the form

$$r' = r(1 + 0{,}5c_{p,\mathrm{v}}\theta_{\mathrm{w}}/r), \tag{1}$$

and as the characteristic dimension use is made of the Laplace constant

$$l = \Lambda = [\sigma/g(\rho' - \rho'')]^{1/2}. \tag{2}$$

The relations in Table 1 follow the equation numbering of the article.

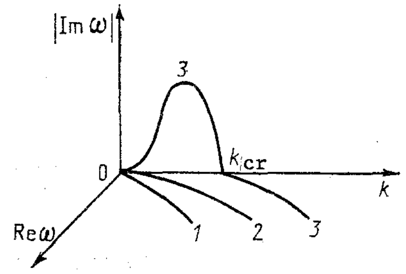

Fig. 2. Dispersion characteristic: 1) $\Delta_f < \Delta_{fm}$; 2) $\Delta_f = \Delta_{fm}$; 3) $\Delta_f > \Delta_{fm}$.

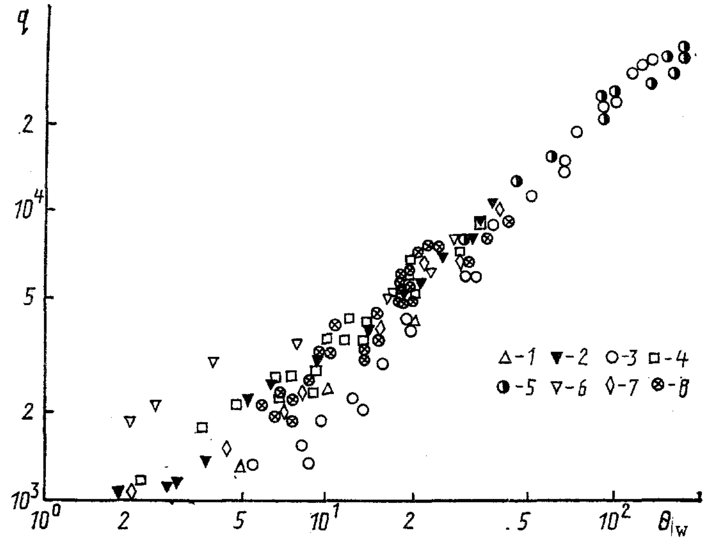

Fig. 3. Film boiling of helium: 1) data from [17]; 2) [22]; 3) [23]; 4) [25]; 5) [27]; 6) [28]; 7) [29]; 8) [30]. q, W/m^2; θ_w, K.

In all the considered studies without exception, the process model in one way or another reflects the already developed instability of the vapor film surface, and the adopted assumptions are an inevitable consequence of the complexity of the original model. The current work uses a new approach, i.e., the heat transfer mechanisms are related directly to the condition of interface stability loss. Such a method was employed for the first time in analyzing film boiling on an extended vertical surface [10]. Its essence lies in identifying the limiting thickness of a laminar vapor film, with which the interface continuity is not yet broken.

Using a perturbation method we define the condition of stability loss as an attainment of a positive value by the resulting perturbation of the forces acting on the interface, which is stipulated by a perturbation of the vapor film thickness

$$\Sigma \Delta P_i \geqslant 0. \tag{16}$$

The following forces act on unit area of the interface:

gravitational force (without regard to the constant component)

$$P_g = g(\rho' - \rho_v)\Delta_f, \tag{17}$$

capillary force

$$P_\sigma = -2\sigma/R \tag{18}$$

and the reaction of a liquid evaporating at the interface

$$P_r = 0{,}5\rho'' v_v^2. \tag{19}$$

We will assume oscillations of the vapor film surface to be sinusoidal, of a wavelength equal to the most probable wavelength of instability according to Taylor [1] (Fig. 1):

$$\lambda_m = 2\pi \sqrt{3}\Lambda. \tag{20}$$

Then, for the wave surface with a perturbation amplitude of the vapor film thickness δ_0

$$\delta = \delta_0 \sin\left(\frac{2\pi x}{\lambda_m}\right) \tag{21}$$

the maximal curvature is defined by the expression

$$\frac{1}{R} = \left|\frac{\partial^2 \delta}{\partial x^2}\right| = \frac{4\pi^2 \delta_0}{\lambda_m^2} = \frac{g(\rho' - \rho'')\delta_0}{3\sigma}. \tag{22}$$

A perturbation of the capillary force is determined by taking into account that the curvature for an unperturbed interface is zero, i.e., 1/R = 0:

$$\Delta P_\sigma = -\frac{2}{3} g(\rho' - \rho'')\delta_0. \tag{23}$$

In the laminar vapor film the expression for a lateral vapor velocity at the interface appears as

$$v_{\mathrm{v}} = \frac{q}{\rho'' r'} = \frac{\lambda_{\mathrm{v}} \theta_{\mathrm{w}}}{\Delta_{\mathrm{f}} \rho'' r'} \tag{24}$$

and the reactive force has the form

$$P_{\mathrm{r}} = \frac{\lambda_{\mathrm{v}}^2 \theta_{\mathrm{w}}^2}{2\Delta_{\mathrm{f}}^2 \rho'' r'^2}. \tag{25}$$

Then, a perturbation of the reactive force is

$$\Delta P_{\mathrm{r}} = \frac{\lambda_{\mathrm{v}}^2 \theta_{\mathrm{w}}^2}{2\rho'' r'^2}\left[\frac{1}{(\Delta_{\mathrm{f}} + \delta_0)^2} - \frac{1}{\Delta_{\mathrm{f}}^2}\right] \simeq -\frac{\lambda_{\mathrm{v}}^2 \theta_{\mathrm{w}}^2}{\Delta_{\mathrm{f}\,m}^3 \rho'' r'^2}\delta_0. \tag{26}$$

In conformity with Eq. (17), the gravitational force perturbation is

$$\Delta P_g = g(\rho' - \rho_{\mathrm{v}})[(\Delta_{\mathrm{f}} + \delta_0) - \Delta_{\mathrm{f}}] = g(\rho' - \rho_{\mathrm{v}})\delta_0. \tag{27}$$

Using Eqs. (23), (26), and (27) and the approximate equality $\rho' - \rho_{\mathrm{v}} \simeq \rho' - \rho'' = \Delta\rho$ we write the stability loss condition

$$\frac{\Sigma \Delta P_i}{\delta_0} = g\Delta\rho - \frac{2}{3} g\Delta\rho - \frac{\lambda_{\mathrm{v}}^2 \theta_{\mathrm{w}}^2}{\Delta_{\mathrm{f}\,m}^3 \rho'' r'^2} = 0, \tag{28}$$

whence the limiting thickness of the vapor film corresponding to the stability boundary is

$$\Delta_{\mathrm{f}\,m} = \left(\frac{3\lambda_{\mathrm{v}}^2 \theta_{\mathrm{w}}^2}{g\rho'' \Delta\rho r'^2}\right)^{1/3}. \tag{29}$$

A more general approach to defining stability of the interface between liquids with different densities is known [11, 12]. A solution for the motion equations is sought in the form of a traveling wave, disregarding viscosity terms.

As applied to the considered problem, the method was first employed in [1], but, without reference to one of the basic characteristics of the process, viz., to the effect of the evaporating liquid reaction on the interface.

A sketch of the problem is shown in Fig. 1. The liquid is situated above, and the vapor below. The waves, with an amplitude small in comparison to the vapor film thickness, propagate over the interface surface.

The motion and continuity equations in a two-dimensional approximation have the form

$$\frac{\partial u_j}{\partial x} + \frac{\partial v_j}{\partial y} = 0, \tag{30}$$

$$\frac{\partial u_j}{dt} = -\frac{1}{\rho_j}\frac{\partial P_j}{\partial x}, \tag{31}$$

$$\frac{\partial v_j}{\partial t} = -\frac{1}{\rho_j}\frac{\partial P_j}{\partial y} - g, \tag{32}$$

where j denotes vapor and liquid.

By representing actual parameters of the process in terms of the averaged and pulsatory components, we obtain

$$u_j = \langle u_j \rangle + u_j', \tag{33}$$

$$v_j = \langle v_j \rangle + v_j', \tag{34}$$

$$P_j = P_{\mathrm{b}} - g\rho_j y + P_j'. \tag{35}$$

With the averaged motion ignored,

$$\langle u_j \rangle = 0, \quad \langle v_j \rangle = 0.$$

Substituting Eqs. (33)-(35) into Eqs. (30)-(32) yields the equations for pulsations

$$\frac{\partial u_j'}{\partial x} + \frac{\partial v_j'}{\partial y} = 0, \tag{36}$$

$$\frac{\partial u_j'}{\partial t} + \frac{1}{\rho_j}\frac{\partial P_i'}{\partial x} = 0, \tag{37}$$

$$\frac{\partial v_j'}{\partial t} + \frac{1}{\rho_j}\frac{\partial P_j'}{\partial y} = 0. \tag{38}$$

The solution is sought in the form of traveling wave:

$$u_j' = \Delta U_j(y)\exp[i(kx + \omega t)], \tag{39}$$

$$v_j' = \Delta V_j(y)\exp[i(kx + \omega t)], \tag{40}$$

$$P_j' = \Delta P_j(y)\exp[i(kx + \omega t)], \tag{41}$$

$$\delta = \delta_0 \exp[i(kx + \omega t)]. \tag{42}$$

Here the following boundary conditions are used:
the condition of wall impermeability

$$\Delta V_{\mathrm{v}}|_{y=-\Delta_{\mathrm{f}}} = 0, \tag{43}$$

the condition of limitedness at infinity

$$\Delta V_{\ell}|_{y\to\infty} = 0, \tag{44}$$

the condition of material balance at the interface

$$\rho''\Delta V_{\mathrm{v}}|_{y=0} = \rho'\Delta V_{\ell}|_{y=0}, \tag{45}$$

as well as the equality

$$v_{\ell}'|_{y=0} = \frac{\partial\delta}{\partial t}. \tag{46}$$

The derived solution for lateral velocity pulsations has the form

$$\Delta V_{v}(y) = \frac{i\omega\delta_0}{\mathrm{sh}\,(k\Delta_{f})}\,\frac{\rho'}{\rho''}\,\mathrm{sh}\,[k\,(y + \Delta_{f})], \tag{47}$$

$$\Delta V_{\ell}(y) = i\omega\delta_0 \exp(-ky), \tag{48}$$

whereas, for pressure pulsations, it appears as

$$\Delta P_{v} = \frac{\rho'\rho_{v}}{\rho''}\cdot\frac{\omega^2}{k}\,\frac{\mathrm{ch}\,[k\,(y + \Delta_{f})]}{\mathrm{sh}\,(k\Delta_{f})}\,\delta_0, \tag{49}$$

$$\Delta P_{\ell} = -\rho'\,\frac{\omega^2}{k}\exp(-ky)\,\delta_0, \tag{50}$$

where δ_0 is the amplitude of interface oscillations.

Let us write the balance of pulsatory pressure components at the interface:

$$-g\rho_{v}y|_{b} + P'_{v}|_{b} + P'_{r} = -g\rho' y|_{b} + P'_{\ell}|_{b} - \sigma\frac{\partial^2\delta}{\partial x^2}. \tag{51}$$

An expression for P_r' is found from Eq. (25) similarly to Eq. (26):

$$P'_{r} = \frac{\lambda_{v}^2\theta_{w}^2}{2\rho'' r'^2}\left[\frac{1}{(\Delta_{f} + \delta)^2} - \frac{1}{\Delta_{f}^2}\right] \simeq -\frac{\lambda_{v}^2\theta_{w}^2}{\rho'' r'^2\Delta_{f}^3}\,\delta. \tag{52}$$

Substituting Eqs. (41), (49), (50), and (52) into Eq. (51) and allowing for the apparent equality $y|_b = \delta(x)$, we arrive, after performing manipulations, at

$$\left[\rho' + \frac{\rho'\rho_{v}}{\rho''}\,\mathrm{cth}\,(k\Delta_{f})\right]\omega^2 = -g\Delta\rho k + \frac{\lambda_{v}^2\theta_{w}^2}{\rho'' r'^2\Delta_{f}^3}\,k + \sigma k^3. \tag{53}$$

Thus, the dispersion characteristic appears as

$$\omega = \left[\frac{\sigma k^3 - \left(g\Delta\rho - \dfrac{\lambda_{v}^2\theta_{w}^2}{\rho'' r'^2\Delta_{f}^3}\right)k}{\rho' + \dfrac{\rho'\rho_{v}}{\rho''}\,\mathrm{cth}\,(k\Delta_{f})}\right]^{1/2}. \tag{54}$$

Hence, the interface oscillations are stable (ω is real) for any wavelength with

$$\Delta_{f} < \Delta_{f\,m} = \left(\frac{\lambda_{v}^2\theta_{w}^2}{\rho'' r'^2 g\Delta\rho}\right)^{1/3} \tag{55}$$

In the opposite case, in the wavelength region specified by the constraint $k < k_{cr}$, where

$$k_{cr} = \left[\frac{1}{\sigma}\left(g\Delta\rho - \frac{\lambda_{v}^2\theta_{w}^2}{\rho'' r'^2\Delta_{f}^3}\right)\right]^{1/2}, \tag{56}$$

the frequency ω becomes imaginary, i.e., the wave amplitude increases in time exponentially, and the dispersion characteristic has the form illustrated in Fig. 2. Therefore, assuming that there are waves of arbitrary lengths at the interface, the violation of condition (55) can be identified with instability onset. In this case, the expression for the limiting thickness of the stable vapor film $\Delta_{f\,m}$ (55) coincides with Eq. (29) accurate to a constant multiplier.

Using the Laplace constant as the characteristic dimension, we can represent an expression for the limiting Nu, corresponding to $\Delta_{f\,m}$, as

$$\mathrm{Nu}_m = \frac{\Lambda}{\Delta_{f\,m}} = \left(\frac{g\Lambda^3\rho''\Delta\rho r'^2}{\lambda_{v}^2\theta_{w}^2}\right)^{1/3} = \left[\mathrm{Ar}_*\left(\frac{\mathrm{Pr}_{v}}{K'_{v}}\right)^2\right]^{1/3} = A^{1/3}. \tag{57}$$

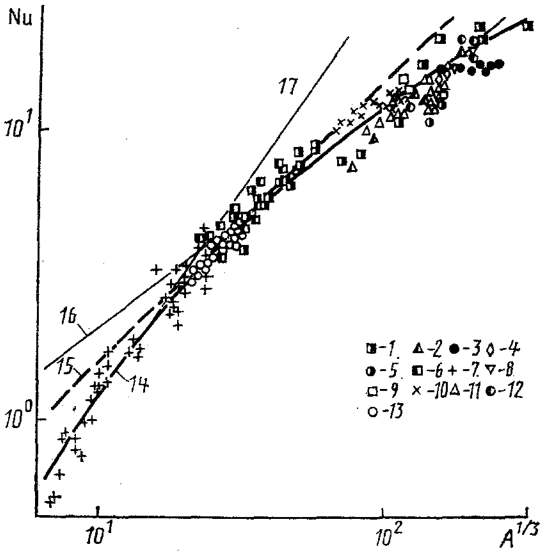

Fig. 4. Generalization of experimental data: 1) nitrogen; 2) argon; 3) acetone; 4) benzene; 5) water; 6) hydrogen; 7) helium; 8) potassium; 9) n-pentane; 10) Freon-11; 11) Freon-113; 12) carbon tetrachloride; 13) ethanol; 14) relation (59); 15) Nu ~ $A^{1/3}$; 16) Nu ~ $A^{1/4}$; 17) Nu ~ $A^{1/2}$.

Let us now address the experimental data. We will restrict ourselves to a self-similar region, with the boundary determined, according to [6], by the following condition for a minimal size of the heat-transfer surface:

$$b_{\min} \geqslant 4\pi\sqrt{6}\Lambda. \tag{58}$$

For example, for water boiling at the atmospheric pressure, $\Lambda = 2.5$ mm and $b_{\min} = 76$ mm.

Hitherto, the bulk of experimental data regarding film boiling on a horizontal surface [5, 13-27] was utilized in [6, 27] for approbating and refining relations (13) and (14). The present study involves, with little exception, the same data and also a number of new ones from [5, 28-32]. In accordance with the recommendations in [31], the experimental data which refer to film-transient boiling, rather than to film boiling, were excluded even though on the boiling curve they are located to the right of the point of minimal temperature head. Furthermore, part of the points for maximal temperature head are eliminated from [24], since a boiling curve inflection, observable in this region, is not physical and can be attributed only to a methodical error of the experiment execution and data processing.

Separate consideration is given to the data for helium, given in Fig. 3. Clearly, with decreasing temperature head a spread in the experimental data becomes larger, amounting to ±70%, therefore, the data for $\theta_w < 20$ K were excluded from subsequent processing, which did not affect the general range of the dimensionless complex.

Table 2 gives brief characteristics of the utilized experimental data. The experiments are performed at atmospheric pressure and at the normal magnitude of acceleration by gravity unless specifically noted.

Although the dimensions of the heat transfer surface for experiments with potassium are not given, it can be deduced from the context of [5] that they also satisfy the condition (58).

Figure 4 shows processing of all experimental points in the coordinates $A^{1/3}$ – Nu. Raising A to the power 1/3 is accomplished not so much for comparing to relation (57) as for "contracting" the abscissa in Fig. 4 (the range of complex A constitutes 5 orders).

If we attempt to approximate the points with a single exponential curve, the exponent 1/3 will indeed be optimal; however, it is seen clearly that a deviation angle of the indicated computational relation coincides with the experimental points only in the middle part of the range of A and differs at the end sections, being smaller in the initial part of the range of A and larger at the end part, as compared to the experimental data. Obviously, the difference from that of "limiting" is explained by the formation of vapor bubbles at the interface, of which no account is taken in determining Nu_m.

TABLE 2. Experimental Data on Film Boiling of a Saturated Liquid on a Horizontal Surface Facing Upwards

Liquid	Surface dimentions, mm	No. of exptl. points	Literature citation	Notes
Nitrogen	44,5×44,5	11	[16]	
Nitrogen	50,8×100	6	[20]	
Nitrogen	Ø50,8	5	[24]	
Nitrogen	Ø40	5	[26]	
Argon	Ø40	5	[27]	
Acetone	Ø63,5	12	[31]	
Benzene	Ø63,5	5	[31]	
Water	Ø101,6	2	[32]	
Hydrogen	25,4×559	14	[13]	$P = (1{,}0\ldots8{,}7)\cdot10^5$ N/m^2
Helium	Ø15,2	1	[17]	
Helium	30×30	5	[22]	
Helium	Ø8	5	[23]	
Helium	Ø16	2	[25]	
Helium	Ø10	30	[27]	
Helium	Ø9,9	2	[28]	
Helium	15×15	3	[29]	
Helium	Ø12	5	[30]	
Ptassium	—	11	[5]	$P = 6{,}6\cdot10^3$ N/m^2
n-Pentene	Ø50,8	7	[14]	
n-Pentene	Ø63,5	11	[31]	
Freon-11	200×200	13	[15]	
Freon-11	75×75	19	[5]	
Freon-113	200×200	9	[19]	
Freon-113	Ø42	16	[21]	
Freon-113	Ø63,5	12	[31]	
Carbon tetrachloride	Ø50,8	4	[14]	
Carbon tetrachloride	200×200	6	[19]	
Ethanol	25×152	122	[18]	$a/g = 3{,}91\ldots21{,}67$

Since the points in Fig. 4 cluster near the curve resembling a logarithmic function, appropriate processing was carried out, and the correlation in the range $A = 3\cdot10^2, \ldots, 3\cdot10^7$ has the form

$$\mathrm{Nu} = 0{,}0022\,\ln^{3{,}3} A = 0{,}034\,\lg^{3{,}3} A. \tag{59}$$

Only 9 of 348 points have a deviation greater than 25% and the value of the standard deviation equals 11%.

It is interesting to remark that the complex in Eqs. (6) and (8), together with others suggested in [2], differs from the complex A solely by the absence of the simplex ρ''/ρ_v; however, the appropriate processing revealed that the approximation error here increases.

Let us attempt to elucidate the character of the derived relation (59) by analyzing the processes of vapor generation and bubble growth.

In the region of large values of the complex A, owing to a relatively high rate of the vapor generation, the time of the bubble inflation up to the separation diameter is governed by the processes of vapor transport in the film, whose magnitude is limited by a vapor viscosity. This viscosity mechanism of film boiling underlies the models treated in [2-4], which lead to the dependence of Nu on the governing complexes, similar in structure to A to power 1/4. As is evident from Fig. 4, such a dependence is indeed characteristic of the indicated region.

Conversely, in the region of small values of the complex A, a relatively high thermal resistance of the vapor film causes the bubble growth time to be determined by a wave parameter, viz., by the buildup time of the oscillation amplitude of the interface after it has lost stability. Such a mechanism of film boiling can be referred to as a wave mechanism.

Should expression (54) be extended to the region of nonlinear perturbations, the characteristic time of exponential growth of the above-stated oscillation amplitude can be obtained approximately assuming that $\Delta_f >> \Delta_{f\,m}$ and $k = k_m = 2\pi/\lambda_m$. Then Eq. (54) results in

$$t_{\mathrm{bub}} \sim \frac{1}{|\mathrm{Im}\,\omega|} \sim \left(\frac{\rho'\Lambda}{g\Delta\rho}\right)^{1/2}. \tag{60}$$

Setting $d_{bub} \sim \Lambda$ in accordance with [1, 33], we acquire an expression for the heat flux removed from a cell adjoining the bubble:

$$Q \sim \frac{r'\rho_v V_{bub}}{t_{bub}} \sim r'\rho_v \Lambda^3 \left(\frac{g\Delta\rho}{\rho'\Lambda}\right)^{1/2}. \tag{61}$$

On the other hand, the heat flux, put by heat conduction through the vapor film into the same cell, is

$$Q_c = \frac{\lambda_v \vartheta_w}{\Delta_f} S_c \sim \frac{\lambda_v \vartheta_w}{\Delta_f} \Lambda^2. \tag{62}$$

The proportionality of S_c and Λ^2 was postulated previously in many studies and verified experimentally, as was the proportionality of d_b and Λ in [32].

Equating Q to Q_c yields

$$r'\rho_v \Lambda^3 \left(\frac{g\Delta\rho}{\rho'\Lambda}\right)^{1/2} \sim \frac{\lambda_v \vartheta_w}{\Delta_f} \Lambda^2, \tag{63}$$

whence

$$\mathrm{Nu} \sim \left(\frac{\rho_v^2}{\rho'\rho''}\right)^{1/2} A^{1/2}, \tag{64}$$

i.e., the relationship satisfactorily describes experimental data in the region of small A to within the constant coefficient and the simplex as is clear from Fig. 4.

Comparing analytic results to the character of the curve in Fig. 4, which approximates experimental points, it can be presumed that the real process incorporates both the examined mechanisms, viz., the wave and viscosity ones, by being their superposition.

Thus, the investigation results provide a possibility of predicting film boiling heat transfer for the considered conditions by a unique relation, over a wide range of the governing complex.

NOTATION

A, A_i, coefficients in Eqs. (5)-(8) and (10); $A = Ar_*(Pr_v^2/K'^2)$; $Ar = gl^3\Delta\rho/\nu_v^2\rho_v$; $Ar_* = Ar(\rho''/\rho_v)$; a, acceleration; b, surface dimension; c_p, specific heat; d, diameter; g, acceleration due to gravity; $K_v = c_{r,v}\theta_w/r$; $K'_v = c_{r,v}\theta_w/r'$; $k = 2\pi/\lambda$, wave number; l, characteristic dimension; Nu, Nusselt number; Nu_m, "limiting" Nusselt number; Q, heat flux; q, heat flux density; R, curvature radius; Ra, Rayleigh number; r, evaporation heat; r′, evaporation heat with consideration of vapor superheating; S, surface area; T, temperature; t, time; ΔU, amplitude of longitudinal velocity; u, longitudinal velocity; V, volume; ΔV, amplitude of lateral velocity; v, lateral velocity; x, longitudinal coordinate; y, transverse coordinate; Δ_f, vapor film thickness; δ, perturbation of vapor film thickness; δ_0, perturbation amplitude of vapor film thickness; $\theta_w = T_w - T_s$; $\Lambda = \sqrt{\sigma/g\Delta\rho}$, Laplace constant; λ, thermal conductivity and wavelength; λ_m, the most probable wavelength of instability according to Taylor; ν, kinematic viscosity; ρ, density; $\Delta\rho = \rho' - \rho''$; σ, surface tension; ω, cyclic frequency; $\langle\ \rangle$, averaged component. Subscripts: b, boundary; l, liquid; v, vapor; bub, bubble; r, reaction; w, wall, c, cell; min, minimal; Σ, total; s, saturation parameter; ′, oscillatory component and saturated liquid; ″, saturated vapor.

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